Clover Math Competition

Form A

April 26, 2024

1. Brain Bean is picking flowers in a field of 35 clovers. Three in every five clovers will always bloom a flower. If he picks the entire field, how many flowers will he have?

2. At the restaurant CloverEats, it costs \$8.50 to order one sandwich and two drinks. It also costs \$11.50 to order two sandwiches and one drink. How many dollars does it cost to order three sandwiches and three drinks?

3. Find the smallest positive integer n such that $n+5$ is divisible by 7 and $n+7$ is divisible by 5.

4. Find the value of $\frac{2^{2024}-2^{2020}}{2^{2020}-2^{2016}}$.

5. Norbit is currently 5 years older than his brother and $\frac{1}{3}$ of his dad's age. Seven years ago, Norbit was twice as old as his brother. How old is his dad today?

6. Square $ABCD$ has side length 5. A line passing through the center of the square divides side AB into two segments of lengths 2 and 3 and divides the square into two regions. Find the positive difference in the areas of the regions.

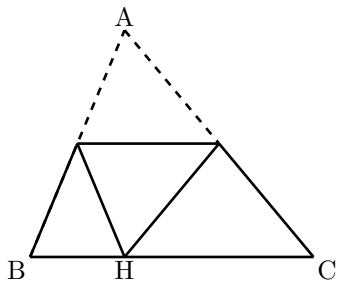
7. Let ω be a rectangular prism whose side lengths are positive integers greater than 1. Given that ω has volume 165, find its surface area.

8. Dheeraj and Krishna start at the same end of a 25-yard swimming pool. Dheeraj swims at a rate of 2 yards per second while Krishna swims at 3 yards per second. When they reach the end of the pool, they turn around and swim in the opposite direction, maintaining a constant speed the whole time. If Krishna and Dheeraj start swimming at the same time, how many seconds pass until they meet each other again?

9. Find the sum $1 \cdot 1! + 2 \cdot 2! + 3 \cdot 3! + 4 \cdot 4! + 5 \cdot 5! + 6 \cdot 6!$.

10. There are 20 numbers that can be created by rearranging the digits in 12225 (including itself). How many of these numbers are divisible by 12?

11. Triangle ABC has side lengths $AB = 13, BC = 15$, and $AC = 14$. Let H be the foot of altitude from A to BC. Fold A down so that A touches H. Find the area of the new figure.



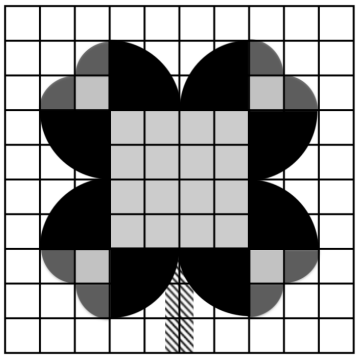
12. The least common multiple of the integers from 1 to 10, inclusive, can be expressed as the prime factorization $2^a 3^b 5^c 7^d$. Find $a+b+c+d$.

13. For some prime number n, the base-n numbers 21_n and 16_n are prime. Find the smallest possible value of n.

14. Jet is removing ducks from a lake using three types of nets. Net A can hold 3 ducks, Net B can hold 5 ducks, and Net C can hold 6 ducks. Using only net types A and B, Jet used 15 total nets to remove all the ducks. Using only nets B and C, Jet used 11 total nets to remove all the ducks, and using only nets A and C, Jet used 10 total nets to remove all the ducks. How many ducks are in the lake?

15. Suppose $\sqrt{4^{4^4}} = 2^{2^{2^x}}$. Find x.

16. Find the area of the clover figure below (excluding the dashed stem) that comprises of unit squares and quarter-circles.

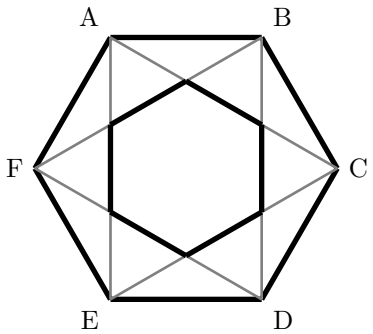


17. A target is made of concentric circles with radii $1, 3, 5, 7$. The inside circle is worth 7 points, the space between the circles of radii 1 and 3 is worth 5 points, the space between the circles of radii 3 and 5 is worth 3 points, and the space between the circles of radii 5 and 7 is worth 1 point. A dart is fired at random at the target and has an equal probability of hitting anywhere on the target. Assuming the dart hits the target, the expected value of the shot is $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

18. Given 6 distinct points, find the largest possible number of triples of points that form an equilateral triangle.

19. Bobby and Joe take turns eating from a bowl of 50 red and 50 blue candy. Bobby always eats one red candy and Joe always eats three candies from whatever color is most abundant. If Bobby starts eating first and they stop eating when they are unable to continue eating, at how many moments are there more blue candies than red candies?

20. In regular hexagon $ABCDEF$, all 9 diagonals are drawn. Together, they bound a smaller hexagon. Given that the larger hexagon has a side length of 6, find the area of the smaller hexagon.



21. What is the maximum number of pieces (of any size) you can cut a pizza into using five linear cuts from your pizza cutter?

22. Jumpy writes a positive integer $n \leq 1000$ on the blackboard. Each second, Jumpy replaces the number on the blackboard according to these rules until the number on the blackboard is 1:

    (a) If the number is even, he replaces it with half of itself.

    (b) If the number is odd, he replaces the number with one more than itself.

    Find the largest number of seconds that will pass before Jumpy reaches 1.

23. Let $a \triangle b = ab - a - b$ and $x * y = xy + y$. Find the number of factors of $27 * (170 \triangle 26)$.

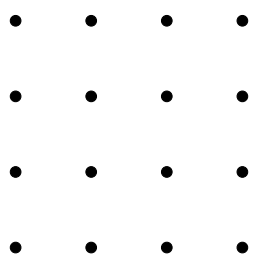
24. A basketball hoop has radius 18 inches and a kid's sized basketball has radius 6 inches. How many of these basketballs can fit into the hoop at once assuming the centers of all the balls lie in the same plane as the hoop?

25. The sequence a_n is defined with $a_1 = 2024$ and for $n \geq 1$, $a_{n+1} = \frac{a_n}{a_n - 1}$. Find a_{2024}.

26. How many triplets of nonnegative integers (x, y, z) satisfy the equation $x^2 + y^2 = 3z^2$?

27. Let n be the number of permutations of the word *cloverleaf* so that no letter is in the same spot when reversed. Find $\frac{n}{6!}$.

28. How many $45 - 45 - 90$ triangles can you form by selecting three points from the following grid?



29. Find the smallest value of n for which n^n is divisible by k^k for at least 10 positive integers k.

30. Define the Fibonacci sequence F_n with $F_1 = 0, F_2 = 1$, and $F_{n+2} = F_{n+1} + F_n$ for $n \geq 1$. Find

$$\sum_{n=1}^{\infty} \frac{F_n}{2^n}.$$